

Year 12 AS/A level Further Maths Induction Task

1 Simplify these expressions as far as possible

a $\frac{x^2 - 2x - 3}{x^2 + 2x + 1}$

b $\frac{x^2 - 25}{x^2 + 6x + 8} \div \frac{x^2 - 2x - 15}{x^2 - 16}$ (7 marks)

2 The line l is a tangent to the circle $x^2 + y^2 = 20$ at the point $P(2, 4)$.

The tangent intersects the y -axis at point A . Find the area of the triangle OPA . (5 marks)

3 Expand and simplify $(\sqrt{p} + 2\sqrt{q})(2\sqrt{p} - \sqrt{q})$ (3 marks)

4 a Write $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$ (3 marks)

b Hence, or otherwise, write down the coordinates of the turning point of the graph

of $y = 3x^2 - 12x + 7$ (1 mark)

5 Prove algebraically that the product of three consecutive **odd** numbers is always an odd number. (4 marks)

6 The functions g and f are defined as $g(x) = \frac{2x}{4 - x}$ and $f(x) = 3x - 1$

Given that $x \neq 4$, find the value(s) of x such that $g(x) = f(x)$, giving your answer(s) to 2 decimal places. (6 marks)

7 The line l_1 has equation $y = -\frac{1}{2}x + 3$ and intersects the x - and y -axes at the points A and B respectively.

a Find the exact length of the line segment AB . (3 marks)

b Find the equation of the line l_2 perpendicular to l_1 which passes through the point $P(-1, -2)$. (2 marks)

The line l_2 intersects l_1 at the point C .

c Find the midpoint of the line segment AC . (4 marks)

- 8 A triangle ABC has side lengths $AB = 10$ cm, $BC = 15$ cm and $AC = 8$ cm.
- a Find the size of the largest angle, giving your answer to 2 decimal places. (3 marks)
- b Find the area of the triangle, giving your answer to 2 decimal places. (2 marks)
- 9 a Sketch the graph of $y = \cos x$ for $-180 \leq x \leq 360^\circ$, showing the points where the graph cuts the axes. (2 marks)
- b Hence find the exact values of x in the interval $-180 \leq x \leq 360^\circ$ for which $\cos x = -\frac{\sqrt{3}}{2}$ (3 marks)